

Evaluation of diffraction errors in precise pulse-echo measurements of ultrasound velocity in chambers with waveguide

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Abstract

Ultrasound velocity measurements in medicine and biology usually are performed using relatively small measurement chambers. When the pulse-echo method is used, the presence of the reflector close to the transducer can cause essential diffraction errors. These errors may be reduced using an additional buffer rod as a waveguide between the transducer and the measurement chamber. The objective of the presented work was analysis of diffraction errors in measurement chambers with a buffer rod.

The work was performed in two steps. In the first stage propagation of transient ultrasonic waves in a buffer rod was analysed using an axisymmetric finite element model. This approach enables all dimensions of the measurement chamber and the waveguide to be taken into account, but is less accurate in the time domain. In the second step the absolute values of diffraction errors were evaluated using a mixed analytic–numeric disk shaped transducer diffraction model. In this case only the dimensions of the waveguide and measurement chamber along the wave propagation direction were taken into account. Diffraction errors were calculated by simulating small changes of ultrasound velocity in the liquid under investigation.

The simulation performed allowed optimisation of the dimensions of the measurement chamber and a buffer rod thus minimising measurement errors. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Ultrasound velocity measurements in medicine and biology usually are performed using relatively small measurement chambers [1,2]. When the pulse-echo method is used, the presence of the reflector close to the transducer can cause essential diffraction errors. These errors may be reduced by using an additional buffer rod as a waveguide between the transducer and measurement chamber and in such a way performing measurements in the far field zone of the transducer. However, when very high accuracy of measurements is required, the influence of diffraction errors may be still unavoidable.

The influence of diffraction phenomena on the accuracy of ultrasonic measurements has been investigated by a number of researchers [3–8]. Most of the publications are devoted to analysis of radiation coupling of disk shaped transducers. However, the majority of the

authors analysed the case of direct coupling of two disk shaped transducers, i.e. when there is no waveguide between the transducers and fluid in which the measurements are performed. The objective of the work presented here was the analysis of diffraction errors of the pulse-echo method in measurement chambers with a buffer rod.

The geometry of the axisymmetric measurement chamber selected for the analysis is presented in Fig. 1. Diffraction errors are caused by deviation of the ultrasonic wave front from a plane wave. Evaluation of the diffraction errors in our case is complicated by the fact that measurements are carried out exploiting reflected ultrasonic waves, which are radiated and received through the waveguide.

If the lateral dimensions of the buffer rod are comparable with the lateral dimensions of the ultrasonic transducer, then in the buffer rod a lot of various modes with different velocities may propagate. Interference of these modes may create a rather non-uniform distribution of displacements at the end of the buffer rod and, thus, affect the ultrasonic field structure in the liquid.

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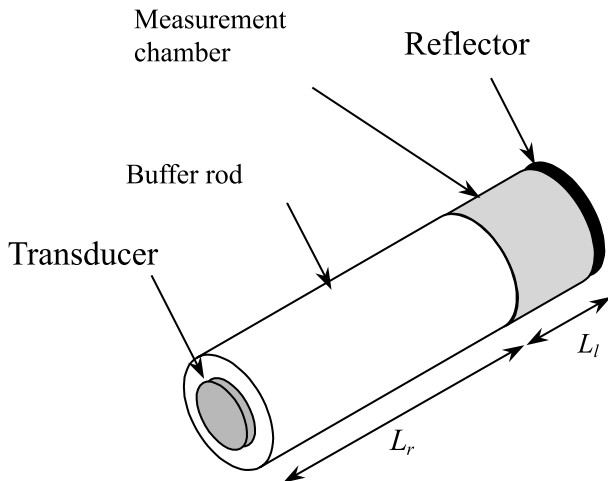


Fig. 1. Schematic presentation of the measurement channel with a buffer rod.

For ultrasound velocity measurements only the fastest mode is exploited, but it is not obvious if the influence of other types of waves is essential, especially when the diameter of the rod is a few times bigger than the diameter of the ultrasonic transducer.

Therefore, the investigation of diffraction phenomena was performed in two steps. In the first stage, propagation of transient ultrasonic waves in a buffer rod was analysed using a finite element model. The aim of this analysis was to determine if it is possible to apply in this case a simpler scalar approach. In the second step, the absolute values of diffraction errors were evaluated using a mixed analytic–numeric model of a disk shaped transducer operating in pulse-echo mode.

2. Finite element simulation

For analysis of a transient ultrasonic field propagating inside the cylindrical buffer rod, the finite element method was applied. This approach enables all finite dimensions of the measurement chamber and the waveguide to be taken into account and all types of waves excited inside the rod to be obtained. Propagation of an elastic wave in a region of arbitrary shape is described by the matrix equation

$$[\mathbf{M}]\{\ddot{\mathbf{U}}\} + [\mathbf{C}]\{\dot{\mathbf{U}}\} + [\mathbf{K}]\{\mathbf{U}\} = \{\mathbf{Q}(t)\}, \quad (1)$$

where $[\mathbf{M}]$, $[\mathbf{C}]$, $[\mathbf{K}]$ are the structural matrices, $\{\mathbf{U}\}$, $\{\mathbf{Q}\}$ are the vectors of nodal displacements and forces in the case of an elastic wave and the vectors of nodal velocity potential and velocities on the boundary in the case of an acoustic wave.

The transient wave analysis was carried out by means of explicit numerical integration of the above equation in the time domain. The main difficulty encountered in treating any real problems of ultrasonic wave propaga-

tion is the huge number of degrees of freedom of the model. This seems to be inevitable because the analysis of wave propagation, as a rule, is being considered in domains, the dimensions of which considerably exceed the wavelength. The number of elements and time steps required is 20–30 elements per wavelength and 20–30 time steps per period of the main harmonic component of the signal.

The main features of the program are the following:

1. Large domains under investigation are subdivided into rectangular subdomains of uniform quadrilateral finite element meshes and into a small number of subdomains of arbitrary geometrical shape represented by free meshes.
2. The domain regularly meshed by quadrilateral elements is subdivided into rectangular subdomains, displacements of which are stored as files on hard disc. For each subdomain the activity index is supplied indicating if the wave front has reached the subdomain. Inactive subdomains are excluded from computation and considerable time saving is achieved during first stages of wave propagation.
3. Products $[\mathbf{K}]\{\mathbf{U}_i\}$ can be evaluated for every individual rectangular subdomain by using displacements of the adjacent zones only.
4. The numerical noise, which “propagates” faster than the longitudinal wave, is eliminated. Checking of the obtained displacement values is performed after every time integration step.

The results of the simulation are presented in Fig. 2. It was assumed that the diameter of the transducer is $D_T = 5$ mm, the diameter and the length of the buffer rod is $D_r = 10$ mm and $L_r = 5$ mm, the ultrasound velocity in the rod $c_r = 2000$ m/s and the density $\rho_r = 1.27 \times 10^3$ kg/m³. The spatial distributions of the longitudinal component of the particle velocity field are shown at subsequent time instants after excitation of the transducer. In the images presented it is possible to observe clearly propagation of the plane longitudinal direct wave and both longitudinal and shear edge waves. In Fig. 3(a) the spatial structure of the ultrasonic field at the instant when the ultrasonic pulse is very close to the end of the buffer rod is presented. In Fig. 3(b) the two-dimensional spatial distribution of the longitudinal component of the particle velocity at the end of the rod at the instant when the ultrasonic pulse crosses the rod–liquid interface is shown.

Measurements of ultrasound velocity in a liquid medium are performed only by means of a longitudinal wave, which, correspondingly, is excited in the liquid only by normal displacement of the end of the buffer rod. From the simulation results, it follows that normal displacement is caused mainly by the direct longitudinal and edge waves. Therefore, when for measurements the

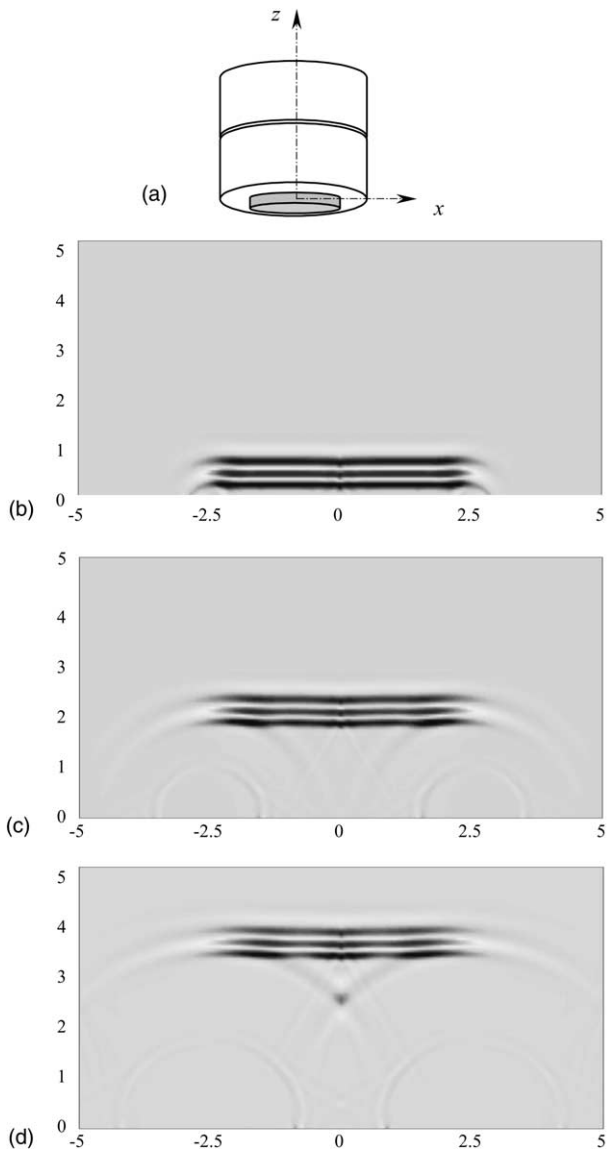


Fig. 2. Propagation of ultrasonic waves in a buffer rod at different time instants: (a) geometry, (b) 609 ns, (c) 1218 ns, (d) 1827 ns.

fastest longitudinal direct and edge waves are used, then for a given geometry a scalar approach taking into account only a longitudinal wave may be applied.

3. Calculation of reflected ultrasonic signals

The absolute values of diffraction errors were evaluated using a mixed analytic–numeric scalar model of an ultrasonic field, radiated and received by a disk shaped transducer. In this case only the dimensions of the waveguide and measurement chamber along the wave propagation direction are taken into account.

An acoustic pressure at an arbitrary point x, y on the reflector surface is found from the convolution of the

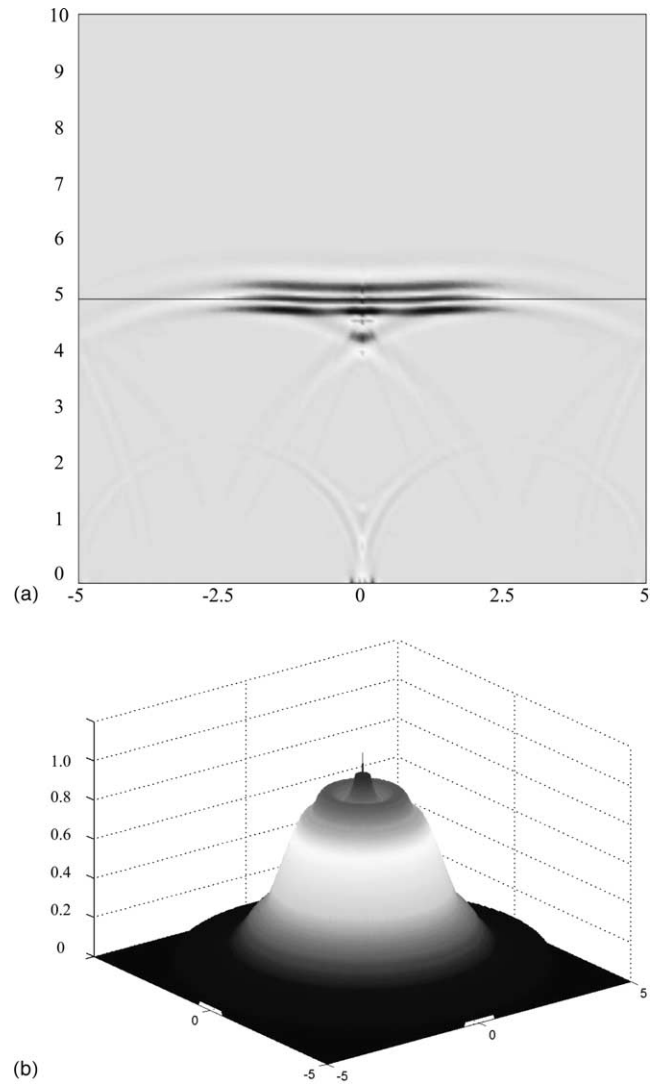


Fig. 3. Ultrasonic field structure (a) and the spatial distribution of the pressure velocity amplitude of the longitudinal component at the second period (b) at the end of the buffer rod at the instant 2436 ns. The spatial position of the end of the buffer rod is denoted by the straight horizontal line.

driving pulse $u(t)$ and the spatial impulse response $h(x, z, t)$:

$$p_a(x, z, t) = k_c \int_0^\infty u(t)h(x, z, t - \tau) dt, \tag{2}$$

where k_c is the constant factor and $h(t)$ is the impulse response of a circular transducer and the waveguide both together.

Due to the reciprocity principle the spatial responses in the transmitting and receiving modes are the same except for the constant factor. Therefore, the signal acting on the surface of the transducer in the receiving mode and caused by a point type reflector located at x, y is given by

$$p(x, z, t) = \int_0^\infty p_a(x, z, t)h(x, z, t - \tau) dt. \tag{3}$$

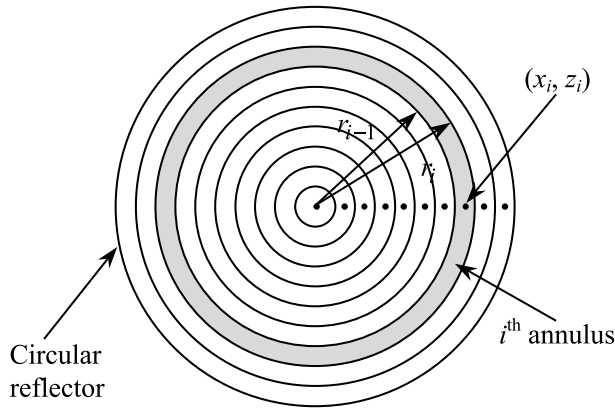


Fig. 4. Division of the circular reflector into elementary annuli.

The acoustic pressure $p(x, z, t)$ is the transducer response in pulse-echo mode for a selected excitation signal. In the case of a circular planar reflector the pressure on the surface of the transducer is found as the sum of the acoustic pulses reflected by annuli of different diameters (Fig. 4):

$$p_r(t) = \pi \sum_{i=1}^M (r_i^2 - r_{i-1}^2) p_i(t, x_i, z_i), \quad (4)$$

where r_{i-1} and r_i are the inner and outer radii of the i th annulus

$$\begin{aligned} r_i &= \left| x_i + \frac{\Delta x}{2} \right| - x_{k0}, \\ r_{i-1} &= \left| x_i - \frac{\Delta x}{2} \right| - x_{k0}, \end{aligned} \quad (5)$$

x_i is the coordinate of the centre line of the annulus and Δx is the width of the annulus, $p_i(t)$ is the signal caused by a point type reflector located at the centre line of the annulus, where $i = 1, \dots, M$ and M is the number of annuli on the reflector surface.

The impulse response $h(t)$ is found by means of the mixed analytic–numeric procedure presented in [9,10]. This approach enables simulation of the ultrasonic field in two media separated by a planar interface. The input parameters for this model are the ultrasound velocities c_1, c_2 and the densities ρ_1, ρ_2 corresponding to the first and second medium, the length of the rod, the distance between the end of the rod and the reflector and the transducer diameter. The driving signal was approximated by

$$u(t) = e^{a(t-b)^2} \sin(2\pi ft), \quad (6)$$

where $a = k_a f \sqrt{-2 \ln 0.1/p_s}$; $b = 2p_s/3f$, p_s is the number of periods, k_a is the asymmetry factor and f is the frequency. Such a signal has the shape of a high frequency ($f = 5$ MHz) pulse with a Gaussian envelope (Fig. 5). The steepness of the front and back slopes of the pulse can be set by separately selecting a corre-

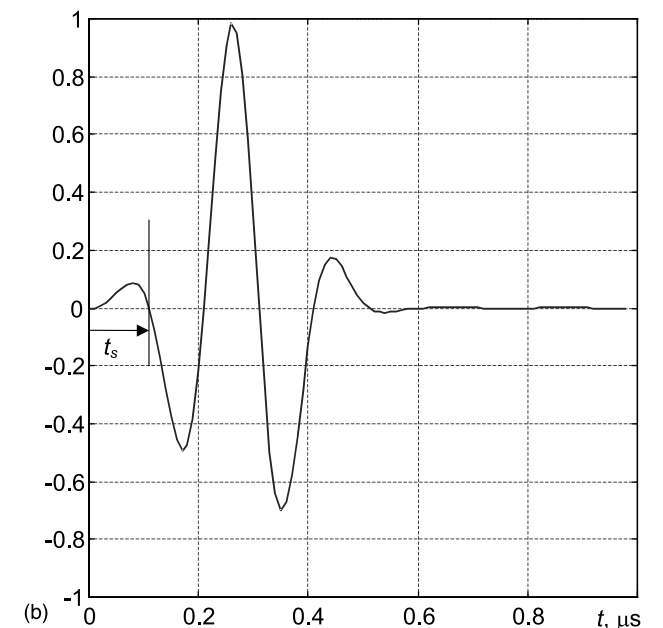
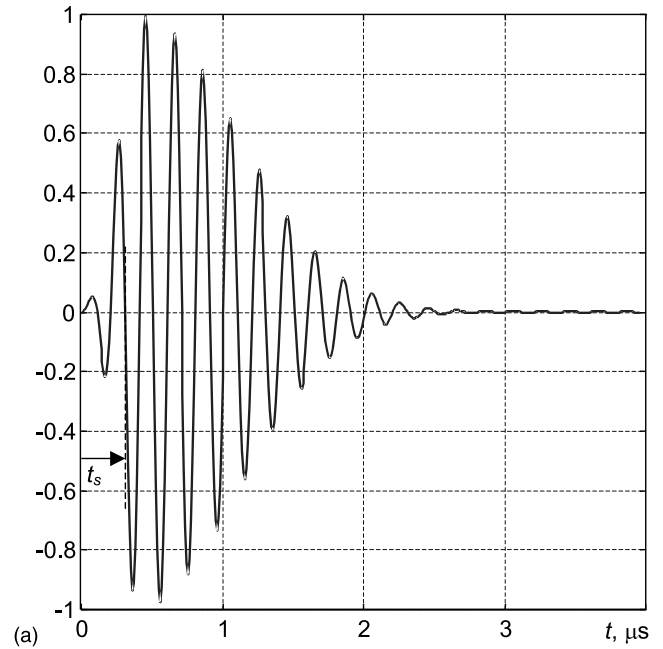


Fig. 5. Waveforms of the driving ultrasonic pulses: (a) $p_s = 10$; (b) $p_s = 1.5$.

sponding value of k_a . Simulation was carried out for two pulses of different duration—short ($p_s = 1.5$) and long ($p_s = 10$), because the duration influences the character of diffraction errors.

Calculations of the transient ultrasonic fields and waveforms in the time domain were performed for an axisymmetric measurement chamber with a plastic buffer rod, the length of which was 15 mm. The circular reflector was placed at the distance 5 mm from the end of the rod. It was assumed that the lateral dimensions of the rod are much bigger than the length. The blood was

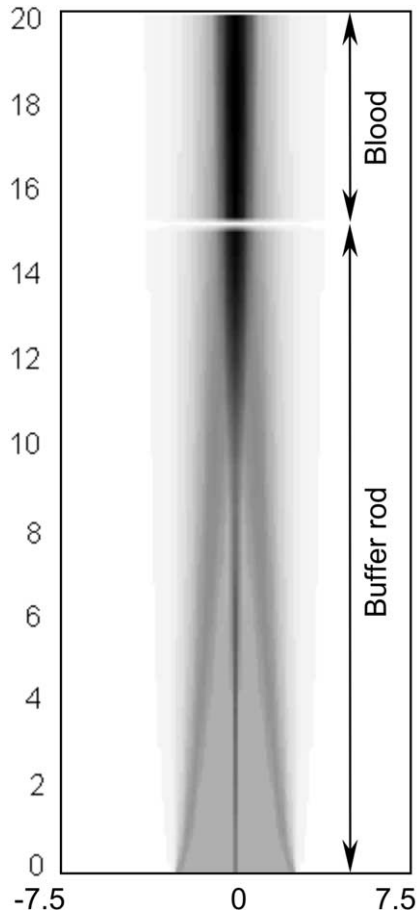


Fig. 6. Ultrasonic field in the pulse-echo mode.

selected as the liquid under investigation with ultrasound velocity $c_b = 1580$ m/s and density $\rho_b = 1.0 \times 10^3$ kg/m³. The calculated pressure field is presented in Fig. 6 as $p_{cs}(x, z) = \max|p(x, z, t)|$. For better understanding, the field presented is normalised with respect to the maximum value of the pressure in the second medium. The spatial distribution of the acoustic pressure across the ultrasonic beam, obtained by this scalar approach, is very close to that obtained by the numerical finite element method (Fig. 7).

4. Diffraction errors

By a diffraction error is understood the difference between the phase velocity of a plane ultrasonic wave and an estimated ultrasound velocity found from the measured delay times of the reflected pulses. The measured ultrasound velocity was obtained from the delay times t_{s1} , t_{s2} of the ultrasonic signals reflected from the end of the buffer rod and the reflector, respectively. These times were estimated as the instant, when the signal at the selected period crosses the zero level. The

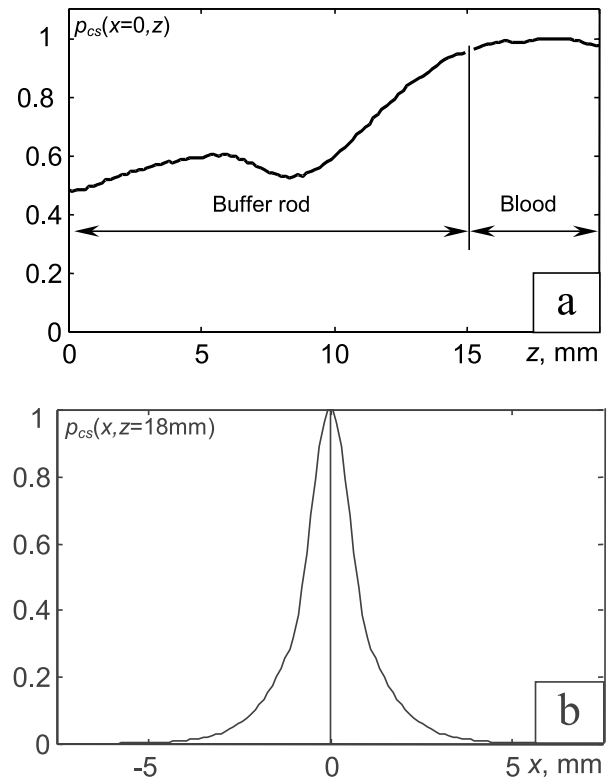


Fig. 7. Spatial distributions of the maximal amplitude of the acoustic pressure along (a) and across (b) the ultrasonic beam.

estimated ultrasound velocity was found using the differential measurement algorithm:

$$\hat{c}_b = \frac{2(z_r - z_w)}{t_{s2} - t_{s1}}. \quad (7)$$

Here z_w and z_r are the distances from the transducer to the end of the buffer rod and the reflector, respectively.

The diffraction error Δc_d was found as the difference between the ultrasound phase velocity value used in calculations c_b and the estimated value \hat{c}_b :

$$\Delta c_d = c_b - \hat{c}_b. \quad (8)$$

Diffraction errors were calculated by simulating small changes of ultrasound velocity in the liquid under investigation. The results of the simulation are shown in Fig. 8. The diffraction errors were calculated both for the long and short driving pulses (Fig. 8(a)). The character of the diffraction errors depends on which zero crossing is used for measurements (Fig. 8(b)). The first zero crossing occurs when the longitudinal direct plane and edge wave arrives, e.g. when the influence of other modes is negligible. The spatial distribution of vibrations on the radiating tip of the rod is the result of interference of the direct plane and edge waves, which is time-dependent. Therefore, at different time instants the field structure in the liquid is also different. For this reason the diffraction errors depend on the number of the zero crossing used in the measurements.

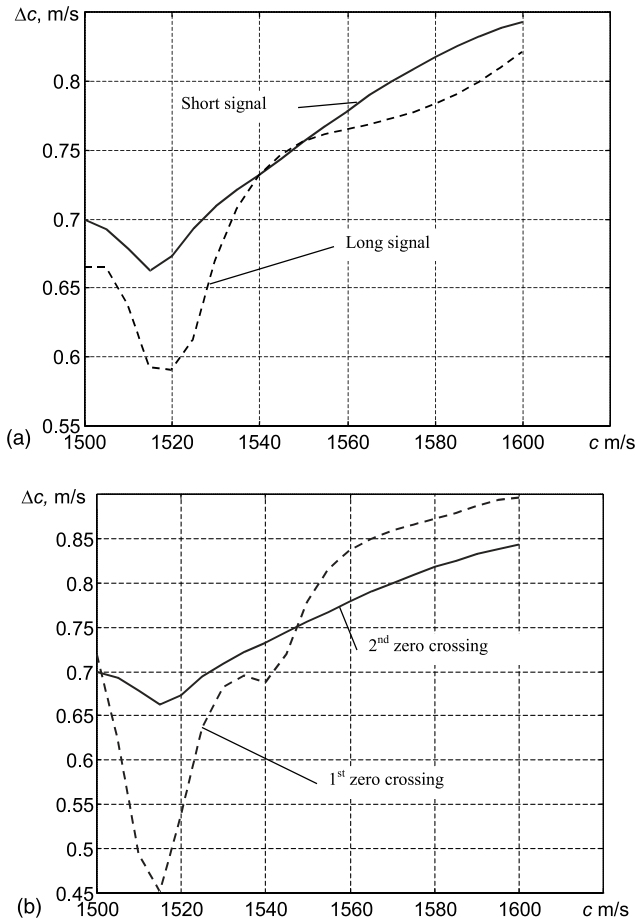


Fig. 8. Diffraction errors versus ultrasound velocity in the object under investigation: (a) for ultrasonic driving pulses of different duration (short signal $p_s = 1.5$ and long signal $p_s = 10$); (b) for different zero crossings.

It is expedient to separate the diffraction error into two components: an absolute error at the fixed ultrasound velocity and a variable part caused by ultrasound velocity variations. The results presented indicate that the absolute diffraction errors in the range of ultrasound velocities 1500–1600 m/s, characteristic for a clotting blood, are of the order of 0.4–0.9 m/s, however their variations are less and do not exceed 0.2–0.4 m/s.

5. Conclusions

The developed analytic–numerical method enables estimation of ultrasound velocity diffraction errors in measurement chambers with a buffer rod. The results obtained showed that their absolute values may reach up to 0.4–0.9 m/s and therefore should be taken into account in precise ultrasound velocity measurements. Optimal selection of the length of the buffer rod and the measurement chamber enables reduction of diffraction errors.

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